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## STATIONARY LONG WAVES IN A LIQUID FILM

## ON AN INCLINED PLANE

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The nonlinear equations describing wave flow of a thin liquid film are normally obtained with the aid of assumptions as to the character of the distribution of the transverse velocity component over the film thickness. Such an approach was used with a system of two equations for liquid flow rate and film thickness deviation from the value corresponding to nonwave laminar flow in [1-3]. In [4-6] a unique evolution equation for film thickness was also obtained with a method similar to the conventional Karman-Polhausen technique. In this case the question of the range of applicability of the equation obtained and the accuracy of its description of the wave process arises. To answer this question one must obviously use direct methods to derive the evolution equation, with simultaneous definition of the velocity profile within the film [7-9]. This will be done below for small Reynolds numbers for a flow on an inclined plane (considered previously in [6, 10, 11]). One of the equations obtained is suitable for study of slightly nonlinear stationary traveling waves. In contrast to previous studies of stationary regimes, all parameters of such waves are defined uniquely.

1. Flow in the Film. We introduce the dimensionless variables and parameters

$$t = \frac{u_0}{\lambda} t', \ x = \frac{x'}{\lambda}, \ y = \frac{y'}{h_0}, \ \begin{cases} v_x \\ v_y \end{cases} = \frac{1}{u_0} \begin{cases} v_x \\ v_y \end{cases},$$

$$\varphi = \frac{h - h_0}{h_0}, \ p = \frac{\text{Re}}{\rho u_0^2} p'_s \text{Re} = \frac{u_0 h_0}{\nu}, \ \varepsilon = \frac{h_0}{\lambda},$$

$$T = \frac{3\varepsilon^3 \text{ We}}{\cos \alpha}, \ \text{We} = \frac{\sigma}{\rho g h_0^2}, \ u_0 = \left(\frac{\cos \alpha}{3} \frac{g}{\nu}\right)^{1/s} Q^{2/s}, \ h_0 = \left(\frac{3}{\cos \alpha} \frac{vQ}{g}\right)^{1/s}.$$
(1.1)

Here the primes denote the corresponding dimensional variables,  $\alpha$  is the angle of inclination of the plate to the vertical,  $\lambda$  is the characteristic longitudinal scale,  $u_0$  and  $h_0$  are the mean film velocity and thickness in the nonwave regime. The equations describing the motion written in the variables of Eq. (1.1) have the form

$$\varepsilon \operatorname{Re}\left(\frac{\partial v_{x}}{\partial t} + v_{x}\frac{\partial v_{x}}{\partial x} - \int_{0}^{y} \frac{\partial v_{x}}{\partial x} dy \frac{\partial v_{x}}{\partial y}\right) = \frac{\partial^{2} v_{x}}{\partial y^{2}} + \varepsilon^{2} \frac{\partial^{2} v_{x}}{\partial x^{2}} - (1.2)$$

$$-\varepsilon \frac{\partial p}{\partial x} + 3_{z} \varepsilon \operatorname{Re}\left(\frac{\partial v_{y}}{\partial t} + v_{x}\frac{\partial v_{y}}{\partial x} - \int_{0}^{y} \frac{\partial v_{x}}{\partial x} dy \frac{\partial v_{y}}{\partial y}\right) = \frac{\partial^{2} v_{y}}{\partial y^{2}} + \varepsilon^{2} \frac{\partial^{2} v_{y}}{\partial x^{2}} - \frac{\partial p}{\partial y} - 3 \operatorname{tg} \alpha, \quad \frac{\partial v_{y}}{\partial y} = -\varepsilon \frac{\partial v_{x}}{\partial x}.$$

The boundary conditions for Eq. (1.2) at y = 0 have the form

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$$v_x = v_y = 0; \tag{1.3}$$

and at  $y = 1 + \varphi$ ,

$$\left(\frac{\partial v_x}{\partial y} + \varepsilon \frac{\partial v_y}{\partial x}\right) \left[1 - \varepsilon^2 \left(\frac{\partial \varphi}{\partial x}\right)^2\right] - 4\varepsilon^2 \frac{\partial v_x}{\partial x} \frac{\partial \varphi}{\partial x} = 0,$$

$$- p \left[1 + \varepsilon^2 \left(\frac{\partial \varphi}{\partial x}\right)^2\right] - 2\varepsilon \frac{\partial v_x}{\partial x} \left[1 - \varepsilon^2 \left(\frac{\partial \varphi}{\partial x}\right)^2\right] - 2\varepsilon \left(\frac{\partial v_x}{\partial y} + \varepsilon \frac{\partial v_y}{\partial x}\right) \frac{\partial \varphi}{\partial x} = \frac{T}{\varepsilon} \left[1 + \varepsilon^2 \left(\frac{\partial \varphi}{\partial x}\right)^2\right]^{-1/2} \frac{\partial^2 \varphi}{\partial x^{2n}}, \quad v_y = \varepsilon \left(\frac{\partial \varphi}{\partial t} + v_x \frac{\partial \varphi}{\partial x}\right).$$

$$= ddition, we will complex the following concentration of the continuity equation.$$

$$(1.4)$$

In addition, we will employ the following consequence of the continuity equation

$$\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial x} \int_{0}^{1+\varphi} v_{x} dy = 0.$$
(1.5)

Usually Eq. (1.5) is considered together with the first equation of Eq. (1.2) integrated over film thickness; sometimes the integral of  $v_X$  is eliminated from this system of two equations. In any case information on the dependence of  $v_X$  upon y is required.

Below we will consider the long wave parameter  $\varepsilon$  to be small, and solve Eqs. (1.2)-(1.4) by the small parameter method. To do this it is necessary to take  $\varepsilon \text{Re} \ll 1$ , i.e., the Reynolds number cannot be too large. In the general case the quantity  $\varphi$  and the parameters  $\varepsilon$  tg  $\alpha$  and T may be of the order of unity, which permits consideration of the flow of liquids with high surface tension over planes slightly inclined to the horizon-tal, where a significant contribution to wave formation can be produced by gravitational waves. (For example, for water at  $\cos \alpha \sim 1$  and  $\text{Re} \sim 1$  we have  $T \sim 10^4 \varepsilon^3$ , so that for real long waves the parameter T is not necessarily always small.)

Taking

$$\begin{cases} v_x \\ v_y \end{cases} = \sum_{n=0}^{\infty} \varepsilon^n \begin{cases} v_x^{(n)} \\ v_y^{(n)} \end{cases}, \ p = \frac{p^*}{\varepsilon} + \sum_{n=0}^{\infty} \varepsilon^n p_n$$

we obtain the zeroth approximation to the problem of Eqs. (1.2)-(1.4):

$$v_x^{(0)} = \left(3 + T\frac{\partial^3 \varphi}{\partial x^3}\right) \left(1 + \varphi - \frac{y}{2}\right) y, \ v_y^{(0)} = 0,$$

$$p^* = -T\frac{\partial^2 \varphi}{\partial x^2}, \ p_0 = 3 \operatorname{tg} \tilde{\alpha} \left(1 + \varphi - y\right).$$
(1.6)

Considering terms on the order of  $\varepsilon$  in Eqs. (1.2)-(1.4) and using Eq. (1.6), we obtain and solve the first approximation problem. We will present only the expression for  $v_x^{(1)}$ :

$$v_x^{(1)} = -\left[3\operatorname{tg}\alpha\frac{\partial\varphi}{\partial x}\eta + \operatorname{Re}\sum_{i=2}^5 \frac{V_i}{i}\eta^i\right]y + \frac{3}{2}\operatorname{tg}\alpha\frac{\partial\varphi}{\partial x}y^2 + \operatorname{Re}\sum_{i=2}^5 \frac{V_i}{i(i+1)}y^{i+1}.$$
(1.7)

In Eq. (1.7) we have introduced the following functions of  $\varphi$  and its derivatives:

$$V_{2} = \frac{\partial F}{\partial t}, \quad V_{3} = \frac{1}{2} \left( H \frac{\partial F}{\partial x} - T \frac{\partial^{4} \varphi}{\partial t \partial x^{3}} \right), \quad (1.8)$$

$$V_{4} = -\frac{1}{3} TG, \quad V_{5} = \frac{1}{12\eta} TG, \quad \eta = 1 + \varphi,$$

$$F = 3\varphi + T \frac{\partial^{3} \varphi}{\partial x^{3}} \eta, \quad H = \left( 3 + T \frac{\partial^{3} \varphi}{\partial x^{3}} \right) \eta, \quad G = H \frac{\partial^{4} \varphi}{\partial x^{4}}.$$

The expressions for  $v_X^{(0)}$  and  $v_X^{(1)}$  in Eqs. (1.6), (1.7) define the velocity profile in the film in the first approximation in terms of the long-wave parameter. This profile differs quite significantly from the parabolic one normally postulated.

2. Evolution Equations. Now calculating the integral of Eq. (1.5), we have

$$\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial x} \left[ \left( 1 + \frac{1}{3} T \frac{\partial^3 \varphi}{\partial x^3} \right) \eta^3 \right] - \varepsilon \frac{\partial}{\partial x} \left[ \operatorname{tg} \alpha \frac{\partial \varphi}{\partial x} \eta^3 + \operatorname{Re} \left( \frac{5}{24} V_2 + \frac{3}{20} V_3 \eta + \frac{7}{60} V_4 \eta^2 + \frac{2}{21} V_5 \eta^3 \right) \eta^4 \right] = 0, \quad (2.1)$$

where the values of  $V_i$  are defined by Eq. (1.8).

Limiting our study to slightly nonlinear waves ( $\psi \ll 1$ ), from Eq. (2.1) we can easily obtain various special variants of the evolution equation. It is of great importance that we immediately consider the relation-

ship between the small parameters  $\varepsilon$  and  $\varphi$ . For example, if  $\varepsilon \operatorname{Re} \lesssim \varphi^2$ , then at  $\varepsilon \operatorname{tg} \alpha \sim 1$  and  $T \sim 1$ , by replacing the derivative  $\partial/\partial t$  in the higher order terms of Eq. (2.1) by  $-3\partial/\partial x$  in accordance with the linearized form of Eq. (2.1), we obtain

$$\frac{\partial \varphi}{\partial t} + 3 (1+\varphi)^2 \frac{\partial \varphi}{\partial x} + \frac{\partial}{\partial x} \left[ (1+3\varphi+3\varphi^2) \left( \frac{1}{3} T \frac{\partial^3 \varphi}{\partial x^3} - \varepsilon \operatorname{tg} \alpha \frac{\partial \varphi}{\partial x} \right) \right] + \varepsilon \operatorname{Re} \left( \frac{6}{5} \frac{\partial^2 \varphi}{\partial x^2} + \frac{15}{56} T \frac{\partial^5 \varphi}{\partial x^5} \right) = 0.$$
(2.2)

But the equations obtained are generally inapplicable at  $\varepsilon \operatorname{Re} \sim \psi$ . In fact, use of Eq. (2.2) with consideration of terms third order in  $\psi$  would imply an increase in accuracy in this case. Moreover, in this case there is no guarantee that the terms of order ( $\varepsilon \operatorname{Re}$ )<sup>2</sup> $\psi$ , which were not considered in Eq. (2.1), are in fact smaller than the retained terms which are third order in  $\psi$  and zeroth order in  $\varepsilon$ . These simple considerations are quite often ignored in film flow studies.

In the case where  $\varepsilon$  tg  $\alpha \ll 1$  and T  $\ll 1$ , from Eq. (2.2) we obtain the equation

$$\frac{\partial \varphi}{\partial t} + 3 \left(1+\varphi\right)^2 \frac{\partial \varphi}{\partial x} + \varepsilon \left(\frac{6}{5} \operatorname{Re} - \operatorname{tg} \alpha\right) \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{3} T \frac{\partial^4 \varphi}{\partial x^4} = 0, \qquad (2.3)$$

which transforms to the well-known equation, if we neglect terms of order  $\varphi^3$ . It is obvious that this is adequate if  $(\epsilon \operatorname{Re})^2 \ll \varphi \ll (\epsilon \operatorname{Re})^{1/2}$ . If the first inequality is not satisfied, then together with terms of order  $\varphi^2$ one must consider terms of order  $\epsilon^2 \varphi$  which were omitted in Eqs. (2.1)-(2.3), while when the second inequality is not satisfied in Eq. (2.3) terms  $\sim \varphi^3$  cannot be neglected.

3. Stationary Wave Regime. For simplicity, we will consider stationary traveling waves only for tg  $\alpha \lesssim 1$ ,  $T \lesssim \epsilon$ . As is well known [12] (but not always considered in film flow studies), in analysis of a wave regime which is close to harmonic, to the accuracy of terms quadratic in the amplitude of the fundamental, one must retain in the evolution equation all terms up to third order in the amplitude. Therefore, we will use an equation in the form of Eq. (2.3). Choosing the longitudinal scale equal to  $h_0$  ( $\epsilon = 1$ ), we rewrite Eq. (2.3) in the form

$$\frac{\partial \varphi}{\partial t} + 3 (1+\varphi)^2 \frac{\partial \varphi}{\partial x} + A \frac{\partial^2 \varphi}{\partial x^2} + B \frac{\partial^4 \varphi}{\partial x^4} = 0, \quad A = \frac{6}{5} \operatorname{Re} - \operatorname{tg} \alpha, \quad B = \frac{\operatorname{We}}{\cos \alpha}.$$
(3.1)

If a stationary periodic regime is established in the system, characterized by a wavelength  $\lambda$ , then we have

$$\varphi = \sum_{n=-\infty}^{\infty} \Phi_n e^{in(\omega t - kx)}, \quad \omega = \Omega - i\gamma, \quad k = 2\pi \frac{h_0}{\lambda}, \quad (3.2)$$

where k,  $\Omega$ , and  $\lambda$  are real, and the fact that  $\psi$  is real gives  $\Phi_{-n} = \Phi_n^*$ , where the asterisk denotes the complex conjugate. Limiting ourselves to terms of order  $q = \Phi_1 \Phi_{-1} = |\Phi_1|^2$ , we consider in Eq. (3.2) only harmonics with  $|n| \leq 2$ . From Eqs. (3.1), (3.2) we obtain

$$[i(\omega - 3k) + Bk^4 - Ak^2] \Phi_1 - 6ik\Phi_0\Phi_1 - 6ik\Phi_{-1}\Phi_2 - 3ik\Phi_{-1}\Phi_1^2 = 0,$$
  

$$[i(\omega - 3k) + 8Bk^4 - 2Ak^2] \Phi_2 - 3ik\Phi_1^2 = 0.$$
(3.3)

From this it follows that

$$i(\omega - 3k) + Bk^4 - Ak^2 + \left[\frac{18k^2}{i(\omega - 3k) + 8Bk^4 - 2Ak^2} - 3ik\right]q - 6ik\Phi_0 = 0.$$
(3.4)

This last equation defines  $\omega$  (and consequently, both  $\Omega$  and  $\gamma$ ) as a function of the dimensionless wave number k and the square of the amplitude q. As  $q \rightarrow 0$ , from Eq. (3.4) we obtain the familiar dispersion equation, following from linear theory (we use  $\Phi_0 \sim q$ ). In the case considered, considering the smallness of q, we write  $\Omega = \Omega_1(k) + \Omega_2(k)q$ ,  $\gamma = \Gamma_1(k) + \Gamma_2(k)q$ .

The stationary regime obviously corresponds to a zero oscillation increment; from this we have  $\gamma = 0$  for the two unknowns k and q. We obtain the second equation from the requirement that this value correspond to the maximum value of  $\gamma$ , considered as a function of k for fixed q [12]. This requirement is also equivalent to the condition of maximum q considered as a function of k. Thus, we have two equations defining k and q:  $\Gamma_1(k) + \Gamma_2(k)q = 0$ ,  $d\Gamma_1(k)/dk + d\Gamma_2(k)q/dk = 0$ .

The calculations can be simplified by noting that the value of  $\Omega$  differs little from the value  $\Omega_0 = 3k$ , obtained from linear theory. This permits neglect of the quantity  $i(\Omega - 3k)$  in the term in Eq. (3.4) proportional to q, so that to the selected accuracy we obtain

$$\Gamma_1(k) = Ak^2 - Bk_1^4 \Gamma_2(k) = 18(A - 7Bk^2)^{-1}$$

and the equations for k and q take on the form

$$q = (k^2/18)(A - Bk^2)(7Bk^2 - A),$$
  
(A - 2Bk<sup>2</sup>)(7Bk<sup>2</sup> - A) + 7Bk<sup>2</sup>(A - Bk<sup>2</sup>) = 0. (3.5)

The solution of the second equation of Eq. (3.5) has the form

$$k = \left(\frac{16 + \sqrt{172}}{42} \frac{A}{B}\right)^{1/2} \approx 0.833 \left(\frac{A}{B}\right)^{1/2}$$
(3.6)

so that the square of the amplitude

$$q \approx 0.046A^3/B. \tag{3.7}$$

Taking  $\Phi_1 = \sqrt{q}$ , which can always be done by appropriate choice of the origin in time or longitudinal coordinate, from the second equation of Eq. (3.3) we obtain an expression for  $\Phi_2$ . The value of  $\Phi_0$  is defined from the condition that the dimensionless liquid flow rate in the film must equal its specified value (unity in the variables of Eq. (1.1)) even in the presence of waves. As a result, to the same accuracy, with consideration of Eqs. (3.6), (3.7), we obtain

$$\Phi_{\mathbf{p}} \approx -2q, \ \Phi_{\mathbf{2}} \approx 0.935 B^{1/2} q i / A^{3/2} \approx 0.043 A^{3/2} i / B^{1/2}. \tag{3.8}$$

Therefore, according to Eqs. (3.4), (3.8) the wave velocity has the form

$$c = \Omega/k = 3(1 - 3q) \approx 3(1 - 0.138A^3/B), \tag{3.9}$$

and the quantity  $\psi$  takes on the following final form:

$$\varphi \approx -0.092 \frac{A^3}{B} + 0.429 \frac{A^{3/2}}{B^{1/2}} \cos k \left(x - ct\right) + 0.086 \frac{A^{3/2}}{B^{1/2}} \sin 2k \left(x - ct\right),$$

with k and c being defined by Eqs. (3.6), (3.9).

We note that k from Eq. (3.6) differs significantly from the value  $k_0 = (A/2B)^{1/2} \approx 0.707 (A/B)^{1/2}$ , which follows for maximum growth waves from linear theory. The condition for instability of nonwave laminar flow has the form A > 0. When it is satisfied a ''soft'' type of stability loss occurs: with increase in the ''super-criticality'' of A the value of q increases monotonically from zero [12, 13]. The stationary wave regime considered proves to be completely defined, in contrast, for example, to the results of [1, 2].

The conditions  $\varepsilon \ll 1$  and  $q^{1/2} \ll 1$  have the respective forms  $(A/B)^{1/2} \ll 2\pi \sim 10^{1/2} - 10$  and  $A(A/B)^{1/2} \ll 10^{1/2} - 10$ . The condition  $\varepsilon \lesssim q$ , which allows us to neglect (at Re ~ 1) all terms except linear ones in the portion of Eq. (2.3) which is proportional to  $\varepsilon$ , now takes on the form  $A^3/B \gtrsim (A/B)^{1/2}$ , i.e.,  $A^{5/2} \gtrsim B^{1/2}$ . If with slightly supercritical A the parameter B is much greater than  $A^5$  (as must be the case for liquids with high surface tension or under lowered gravitation), terms quadratic in  $\varphi$  must be considered in this portion of the equation. In conclusion, we note that attempts (see, for example, [11, 14]) to obtain the coefficients for the higher harmonics in Eq. (3.2) by analysis of equations of the form of Eq. (3.1) are meaningless, since the accuracy of the equation is exceeded.

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EXPERIMENTAL STUDY OF BUOYANT VORTEX RINGS

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UDC 532.517.4

Experimental study of the dynamics and the internal structure of buoyant vortex-thermals was initiated in the 1950s. A review of work done outside the Soviet Union is given in [1]. Problems associated with the motion of thermals are discussed in Soviet publications [2-5]. Experimental data from early as well as those obtained from later studies do not cover many aspects of these phenomena and even contradict each other in some cases. The objective of the present investigations is to study as completely as possible the motion of thermals for various values of initial weight defect. The experimental setup and the test results are described in this paper.

1. The setup is shown in Fig. 1. It consists of: a  $1.2 \times 1.2 \times 5$  m airtight container with transparent side walls; a device (D) for producing thermals; pneumatic and measurement systems. The last mentioned consists of hot-wire anemometers (DISA), movie-(K) and photographic camera, stroboscope, and also an apparatus for visualization of thermals.

The device to produce thermal consists of a nozzle with a funnel shaped tip 4 to blow soap bubble and a trigger to puncture the membrane.

The nozzle is connected by a rubber hose to the smoke generator  $(G_d)$  and further to the mixer (M) where helium, nitrogen, and oxygen were fed from tanks. The partial pressures of each gas in the mixer were measured with the manometer MVP-2.5 which enabled the determination of density with less than 1% error.

The density was varied in the tests by altering the proportions of helium and nitrogen. Oxygen constituted 2.5% by volume in all the tests. It was supplemented by tobacco smoke which was introduced into the soap bubble for coloring it. A small amount of oxygen ensured low smokiness and, consequently, small relative errors in the final determination of the initial density of the thermal.

The trigger consists of an electromagnet 2, a spoke 3 with a needle at its end.

The film is ruptured when the needle punctures the bubble during its descent which began after the application of the starting voltage to the electromagnet and lasted about 0.02 sec.

Besides the device for producing thermals (setup at the bottom of the container) there is a vertical support 5 to which thin streamlined probes 6 are attached at three levels. The anemometer wires are fixed to the probe tips in such a way that their axes coincide with the central axes of the nozzle and the container. Leads from the hot-wires are brought out from the top of the container and connected to hot-wire anemometers whose signals were recorded with automatic recorder H 338.

Flow visualization of the mixture colored by smoke was made with the help of electronic flash ( $F_V$ ) and optical knife laser beam (L) LG-106M which is propagated in a fan shaped manner with the help of the convex mirror 3. The thermal visualized in such a manner was photographed during its motion by the cameras  $F_1$  and

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